



Relativistic Invariance and Commutation Relations at Unitary Quantum Theory

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ABSTRACT

The present article discusses the problems of relativistic invariance and commutation relations at unitary quantum theory. The scalar analogue of the main (principal) equation of the unitary quantum theory together with the Poisson equation are solved numerically in this paper. The value of the electrical charge and also the fine-structure constant, are found, which are in good agreement with the experiment. The evaluation of the electrical form factor and the mass of such a particle is also carried out.

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Introduction

The previous investigations [1-4] have suggested a model of the unitary field theory where a particle with mass m is described by the equation

$$i\lambda^\mu \frac{\partial \Phi}{\partial x^\mu} - m\Phi = 0 \quad (1)$$

and each component Φ_s of the wave function satisfies the second order equation

$$u^\mu u^\nu \frac{\partial^2 \Phi_s}{\partial x^\mu \partial x^\nu} + m^2 \Phi_s = 0 \quad (2)$$

so that the commutation relations for matrices λ^μ have the form

$$\lambda^\mu \lambda^\nu + \lambda^\nu \lambda^\mu = 2g^{\mu\nu} I \quad (3)$$

where $x^\mu = (t, \mathbf{x}); u^\mu = (\frac{1}{\gamma}, \frac{\mathbf{v}}{\gamma})$ is the particle velocity; $\mu, \nu = 0, 1, 2, 3$; a metrics with signature (+, -, -, -) is used; c and \hbar equal 1, and repeated indices are assumed to be summed.

Common approach

For equation (1) to be the starting point of the theory, the equation should first result in the correct energy-momentum relation for a free particle and then be the Lorentz covariant. Equation (2) meets the former condition in the form

$$(p^\mu u_\mu)^2 = m^2$$

Matrices are functions of the particle velocity, and thus the commutation relations (3) alone are insufficient for proving invariance of eq. (1) under the Lorentz transformations; therefore let us first specify the functional dependence of the matrices on the velocity. Since the trivial solution

$$\lambda^\mu = u^\mu I$$

is totally uninteresting, let us consider the case of linear dependence on the velocity

$$\lambda^\mu = \lambda^{\mu\sigma} u_\sigma + \lambda^{\mu 4} \quad (4)$$

where $\lambda^{\mu\sigma}$ and $\lambda^{\mu 4}$ are numerical matrices. The condition (3) holds identically if

$$\begin{aligned} \lambda^{\mu\sigma} \lambda^{\nu\tau} + \lambda^{\nu\tau} \lambda^{\mu\sigma} &= 2(g^{\mu\tau} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\tau}) I \\ \lambda^{\mu 4} \lambda^{\nu 4} + \lambda^{\nu 4} \lambda^{\mu 4} &= 2g^{\mu\nu} I \end{aligned} \quad (5)$$

$$\lambda^{\mu 4} \lambda^{\nu\tau} + \lambda^{\nu\tau} \lambda^{\mu 4} = 0$$

Because of the antisymmetry of $\lambda^{\mu\sigma} = -\lambda^{\sigma\mu}$, only ten out of the twenty matrices are independent quantities. These matrices mutually anticommute, the square of four of them is equal to unity and that of six, to minus unity. To put it differently, eq.

(5) is specified by ten generatrices of the alternion algebra ${}^4A_{11}$, which is isomorphous with the algebra of the sixteenth order quaternion matrices [Zaitsev, 1974]. Since they are not convenient, let us replace the quaternion matrices with ten complex, irreducible, unitary 32^{nd} order matrices

$$(\lambda^{\mu\nu})^+ = (\lambda^{\mu\nu})^{-1}, \quad (\lambda^{\mu 4})^+ = (\lambda^{\mu 4})^{-1} \quad (6)$$

This situation arises in construction of Dirac matrices, which are usually chosen as complex fourth order matrices even though the equation

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} I$$

is satisfied by four second-order quaternion matrices.

From eqs. (5) and (6) it follows that four matrices are Hermitian and six are anti-Hermitian

$$(\lambda^{0a})^+ = \lambda^{0a}, \quad (\lambda^{ab})^+ = -\lambda^{ab}, \quad a, b = 1, 2, 3, 4 \quad (7)$$

If a matrix Λ is introduced

$$\Lambda = \lambda^{12} \lambda^{13} \lambda^{14} \lambda^{23} \lambda^{24} \lambda^{34}, \Lambda^+ = \Lambda^{-1} = -\Lambda \quad (8)$$

then the Hermitian conjugations conditions (7) can be rearranged into

$$(\lambda^{\alpha\beta})^+ = \Lambda \lambda^{\alpha\beta} \Lambda^{-1} \quad (9)$$

Represented in the form (5) the commutation relations are unwieldy and inconvenient in proving the relativistic invariance; however, they can be represented in a simpler form. Let us

define a symmetrical tensor $g^{\alpha\beta}$

$$g_{00} = -g_{11} = -g_{22} = -g_{33} = -g_{44} = 1$$

$$g_{\alpha\beta} = 0 \quad \text{if} \quad \alpha \neq \beta \quad (10)$$

henceforth subscripts of initial letters of the Greek alphabet $\alpha, \beta, \gamma, \delta$ take on values from 0 to 4 while those of the middle

of the alphabet from 0 to 3. The inverse tensor $g^{\alpha\beta}$ provides a compact restatement of commutation relation (5)

$$\lambda^{\alpha\beta} \lambda^{\gamma\delta} + \lambda^{\gamma\delta} \lambda^{\alpha\beta} = 2(g^{\alpha\delta} g^{\beta\gamma} - g^{\alpha\gamma} g^{\beta\delta}) I \quad (11)$$

Eqs. (4), (10) and (11) make it possible to prove the relativistic invariance of eq. (1) by using a five-dimensional group of transformations of coordinate $O(4,1)$. For this purpose extend eq. (1) to the case of a five-dimensional pseudo-Euclidian space with a metric tensor (10)

$$i\lambda^{\alpha\beta} u_\alpha \frac{\partial \Phi}{\partial x^\beta} - m\Phi = 0 \quad (12)$$

(where u^α is the 5-velocity, $u^\alpha u_\alpha = 0$) and then prove invariance of this equation under the group of five-dimensional transformation $O(4,1)$, which contains the Lorentz group as a subgroup. Under reduction of $O(4,1)$ to the Lorentz group, we

assume that $x^4 = \text{Const}, u^4 = 1$ and $\frac{\partial}{\partial x^4} \equiv 1$ then we have eq. (1); in other words, one can assume that eq. (1) is invariant under five-dimensional transformations, but the physical solution does not depend on the fifth coordinate. Incidentally, eq. (12) can be interpreted differently, but we will not discuss these possibilities, for using the five dimensions is merely a convenient tool, which enables us to make full use of simplicity of the commutation relations (11).

To prove invariance of the equation, it is sufficient to show [20] that for any transformation of coordinates

$$\begin{aligned} (x^\alpha)' &= a^\alpha_\beta x^\beta; \\ (x^\alpha)' x'_\alpha &= \text{inv} \end{aligned} \quad (13)$$

there is a linear transformation $S(a)$ of wave functions, the primed and unprimed reference frame

$$\begin{aligned} \Phi'(x') &= S(a)\Phi(x), \\ \Phi(x) &= S^{-1}(a)\Phi'(x') \end{aligned} \quad (14)$$

and $\Phi'(x')$ is a solution of the equation, which has the form of eq. (12) in the primed reference frame

$$\left[i\lambda^{\alpha\beta} u'_\alpha \frac{\partial}{\partial (x^\beta)'} - m \right] \Phi'(x') = 0 \quad (15)$$

Substitute (14) into (12); multiply the left-hand side by $S(a)$, and use the definition (13) to have

$$\left[iS\lambda^{\alpha\beta} S^{-1} a^\gamma_\alpha a^\delta_\beta u'_\gamma \frac{\partial}{\partial (x^\delta)'} - m \right] \Phi'(x') = 0$$

This equation coincides with (15), if the matrix has the property

$$a^\gamma_\alpha a^\delta_\beta S \lambda^{\alpha\beta} S^{-1} = \lambda^{\gamma\delta} \quad (16)$$

Construct S for the infinitesimal proper transformation of the group $O(4,1)$

$$\begin{aligned} a^\beta_\alpha &= \delta^\beta_\alpha + \mathcal{E}^\beta_\alpha; \\ a_{\alpha\beta} &= g_{\alpha\beta} + \mathcal{E}_{\alpha\beta} \end{aligned} \quad (17)$$

with

$$\mathcal{E}_{\alpha\beta} = -\mathcal{E}_{\beta\alpha} \quad (18)$$

Expand S in power of \mathcal{E} and keep only linear terms

$$S = 1 - \frac{1}{4} \sigma^{\alpha\beta} \mathcal{E}_{\alpha\beta} \quad (19)$$

where $\sigma^{\alpha\beta} = -\sigma^{\beta\alpha}$ by eq. (18). Substitute eqs. (17)-(19) into eq. (16), keep first-order terms in \mathcal{E} , use the notation [B,C]=BC-CB for the commutation brackets and have

$$2[\sigma^{\alpha\beta}, \lambda^{\gamma\delta}] = g^{\alpha\delta} \lambda^{\beta\gamma} - g^{\alpha\gamma} \lambda^{\beta\delta} + g^{\beta\gamma} \lambda^{\alpha\delta} - g^{\beta\delta} \lambda^{\alpha\gamma}$$

The antisymmetric solution of this equation

$$\sigma^{\alpha\beta} = \frac{1}{2} g_{\gamma\delta} [\lambda^{\beta\gamma}, \lambda^{\alpha\delta}] \quad (20)$$

is, by virtue of diagonality of the metric tensor and antisymmetry of $\lambda^{\alpha\beta}$, a sum of mutually commuting terms; in

particular, σ^{12} has the form

$$\sigma^{12} = \lambda^{20} \lambda^{10} - \lambda^{23} \lambda^{13} - \lambda^{24} \lambda^{14}$$

According to eq. (19) S for an infinitesimal transformation is given by

$$S = 1 - \frac{1}{8} g_{\gamma\delta} \mathcal{E}_{\alpha\beta} [\lambda^{\beta\gamma}, \lambda^{\alpha\delta}]$$

Hence, for rotation through a finite angle ω about this axis in the direction labeled n is represented as

$$S = \exp\left\{ -\frac{1}{4} \omega \sigma^{\alpha\beta} P_{\alpha\beta}^n \right\} \quad (21)$$

where $P_{\alpha\beta}^n$ is the generator of rotation about this axis. The matrix S is not, generally speaking, unitary but formula (9) easily shows that

$$\Lambda^{-1} \sigma^+ \Lambda = -\sigma,$$

consequently, for proper transformations

$$\Lambda^{-1} S^+ \Lambda = S^{-1} \quad (22)$$

Let us consider improper transformations of space reflection and time reversal. For space reflection the matrix a is diagonal

$$a_0^0 = a_4^4 = -a_1^1 = -a_2^2 = -a_3^3 = 1,$$

then eq. (16) for the space reflection operator P is satisfied by

$$P = \lambda^{01} \lambda^{02} \lambda^{03} \lambda^{14} \lambda^{24} \lambda^{34} = P^+ = P^{-1} \quad (23)$$

which ensures invariance of both eq. (1) and eq. (12). Construct a transformation of the time inversion; for this purpose introduce an interaction of a particle whose charge is e with an external

electromagnetic field $A^\mu = (\varphi, A^k)$ by means of the gauge invariant substitution

$$i \frac{\partial}{\partial x^\mu} \rightarrow i \frac{\partial}{\partial x^\mu} - eA_\mu$$

and rewrite eq. (1) in the form [2,3,5]:

$$i\lambda^0 \frac{\partial \Phi}{\partial t} = \left[\lambda^k \left(-i \frac{\partial}{\partial x^k} + eA_k \right) + m + e\varphi \lambda^0 \right] \Phi = H\Phi$$

transformation T as such that if $t' = -t, \Phi'_T = \Phi'(t') = T\Phi(t)$; then the latter equation becomes

$$-(Ti\lambda^0 T^{-1}) \frac{\partial \Phi'(t')}{\partial t'} = (THT^{-1}) \Phi'(t')$$

When the sense of time is reserved

$$u'_0 = u_0; u'_k = -u_k; \Phi' = \Phi; A'^k = -A^k$$

and, before all, it is necessary to change the sign between two

terms $i \frac{\partial}{\partial x^k}$ and eA_k ; therefore the transformation is regarded as a complex conjugation operator multiplied by the matrix T :

$$\Phi'_T = T\Phi(t) = T\Phi^*(t) \tag{24}$$

This gives

$$\left(T \lambda^0 T^{-1} \right) \frac{\partial \Phi'(t')}{\partial t'} = \left\{ -(T \lambda^k T^{-1}) \left[-i \frac{\partial}{\partial (x^k)} + eA_k \right] + m + e\varphi \left(T \lambda^0 T^{-1} \right) \right\} \Phi'(t')$$

and for invariance of the equation it is necessary that

$$T\lambda^0 T^{-1} = \lambda^0; T\lambda^k T^{-1} = -\lambda^k; T\lambda^k T^{-1} = \lambda^k; T\lambda^0 T^{-1} = \lambda^0 \tag{25}$$

Thence it immediately follows that $T^* = T^{-1} = T$, though the explicit form of the matrix T depends on the particular representation of the matrix $\lambda^{\alpha\beta}$. Note that there is just one matrix

$$\lambda = \prod_{\alpha < \beta}^4 \lambda^{\alpha\beta}$$

which commutes with both generators $\sigma^{\alpha\beta}$ for the representation of the group $O(4,1)$ and with the operators of discrete transformation P and T . Under reduction of $O(4,1)$ to the Lorentz group two more matrices

$$\Lambda_1 = \lambda^{04} \lambda^{14} \lambda^{24} \lambda^{34}; \Lambda_2 = \lambda \Lambda_1$$

are generated which commute with the generators $\sigma^{\mu\sigma}$ of the representation of the Lorentz group and anticommute with P and T . Consequently, formulae (21), (23)-(25) specify the reducible representation of the Lorentz group and this representation is double-valued. Indeed, consider a particular case, rotation through angle ω about the Z-axis. In this case $P_{12}^Z = -P_{21}^Z = 1$; using the explicit form of σ^{12} we have

$$S = \exp\left(-\frac{\omega}{2} \sigma^{12}\right) = \cos^3\left(\frac{\omega}{2}\right) + \sigma^{12} \cos^2\left(\frac{\omega}{2}\right) \sin\left(\frac{\omega}{2}\right) + \frac{3 + (\sigma^{12})^2}{2} \cos\left(\frac{\omega}{2}\right) \sin^2\left(\frac{\omega}{2}\right) + \lambda^{20} \lambda^{10} \lambda^{23} \lambda^{13} \lambda^{24} \lambda^{14} \sin^3\left(\frac{\omega}{2}\right)$$

The half-angle is an expression of the double value of the wave function transformation. Therefore the observables in the theory should be bilinear in $\Phi(x)$. The matrix Λ makes it possible to

determine the adjoint wave function $\bar{\Phi} = \Phi^\dagger \Lambda$, which is a solution of the adjoint equation

$$i \frac{\partial \bar{\Phi}}{\partial x^\mu} \lambda^\mu + m \bar{\Phi} = 0$$

An adjoint wave function under an arbitrary transformation of the co-ordinates should be transformed by the equation

$$\bar{\Phi}' = \bar{\Phi} \Lambda^{-1} S^\dagger \Lambda$$

$$\bar{\Phi}' = \bar{\Phi} S^{-1},$$

$$\bar{\Phi}' = -\bar{\Phi} T^{-1},$$

$$\bar{\Phi}' = -\bar{\Phi}^* T^{-1},$$

$$\bar{\Phi}' \Phi'_P = -\bar{\Phi} \Phi$$

$$\bar{\Phi}' \lambda \Phi'_P = -\bar{\Phi} \lambda \Phi$$

$$\bar{\Phi}' \Lambda_1 \Phi'_P = \bar{\Phi} \Lambda_1 \Phi$$

$$\bar{\Phi}' \Lambda_2 \Phi'_P = \bar{\Phi} \Lambda_2 \Phi$$

$$\bar{\Phi}' \Phi'_T = \bar{\Phi} \Phi$$

$$\bar{\Phi}' \lambda \Phi'_T = -\bar{\Phi} \lambda \Phi$$

$$\bar{\Phi}' \Lambda_1 \Phi'_T = -\bar{\Phi} \Lambda_1 \Phi$$

$$\bar{\Phi}' \Lambda_2 \Phi'_T = \bar{\Phi} \Lambda_2 \Phi$$

Following the classification of Ref. [18,20], the quantities (26a-d) are singular and simple pseudo-scalar and singular and simple scalar, respectively, each of these functions being a unique scalar function of the associated type, quadratic in $\Phi(x)$. To obtain a numerical scalar let us use a representation of the function $\Phi(x)$ as a four-dimensional Fourier integral. Since each component of $\Phi(x)$ satisfies the second order equation (2), the general solution represented entirely in relativistic terms has the form

$$\Phi(x) = \frac{2}{(2\pi)^{3/2}} \int d^4 k e^{ik_\mu x^\mu} \delta\left\{ (k_\mu x^\mu)^2 - m^2 \right\} \Phi(k) \tag{27}$$

where

$$\delta\left\{ (k_\mu x^\mu)^2 - m^2 \right\} = \frac{1}{2m} \left\{ \delta(k_\mu x^\mu - m) + \delta(k_\mu x^\mu + m) \right\}$$

is the relativistic δ -function and the amplitude $\Phi(k) = \Phi(k^0, \mathbf{k})$ satisfies the equation $(\lambda^\mu k_\mu + m)\Phi(k) = 0$ for $(ku)^2 = m^2$

Because the integrand includes a δ -function, the integration is performed over just two Lorentz-invariant hyper surfaces $k_\mu u^\mu = \pm m$, rather than the entire four-dimensional k -space. This allows for decomposing the integral (27) into two summands

$$\Phi(x) = \Phi^+(x) + \Phi^-(x);$$

$$\Phi^\pm(x) = \frac{1}{(2\pi)^{3/2}} \int d^4k \frac{\delta(k_\mu u^\mu \mp m)}{2m} \Phi(k) \tag{28}$$

Using this representation and integrating over the three-dimensional volume, we have

$$\int \bar{\Phi}^\pm u^\mu \frac{\partial \Phi}{\partial x^\mu} \frac{dV}{\gamma} = - \int u^\mu \frac{\partial \bar{\Phi}^\pm}{\partial x^\mu} \Phi^\pm \frac{dV}{\gamma} =$$

$$= \pm \frac{i}{2m} \int d^4k \delta(k_\mu u^\mu \mp m) \bar{\Phi}(k) \Phi(k)$$

$$\int \bar{\Phi}^\pm u^\mu \frac{\partial \Phi^\mp}{\partial x^\mu} \frac{dV}{\gamma} = \int u^\mu \frac{\partial \bar{\Phi}^\pm}{\partial x^\mu} \Phi^\mp \frac{dV}{\gamma} =$$

$$= \mp \frac{i}{2m} \int d\mathbf{k} \exp\left(\mp \frac{2ix^0 m}{u^0}\right) \bar{\Phi}\left(\frac{\mathbf{k}u \pm m}{u^0}, \mathbf{k}\right) \Phi\left(\frac{\mathbf{k}u \mp m}{u^0}, \mathbf{k}\right)$$

Combining these relations and using the equality $\delta(k_\mu u^\mu - m) - \delta(k_\mu u^\mu + m) = \theta(k_\mu u^\mu) \delta\left\{(k_\mu u^\mu)^2 - m^2\right\}$ we find that

$$\int \left(\bar{\Phi} u^\mu \frac{\partial \Phi}{\partial x^\mu} - u^\mu \frac{\partial \bar{\Phi}}{\partial x^\mu} \Phi \right) \frac{dV}{\gamma} =$$

$$i \int d^4k \theta(k_\mu u^\mu) \delta\left\{(k_\mu u^\mu)^2 - m^2\right\} \bar{\Phi}(k) \Phi(k) \tag{29}$$

where

$$\theta(ku) = \begin{cases} 1, & \text{if } ku > 0 \\ -1, & \text{if } ku < 0 \end{cases}$$

The right-hand side of eq. (29) is explicitly represented in covariant form, which facilitates a study of properties, which can be traced to the space and time inversions. More specifically,

eq. (29) is a simple pseudo-scalar because $\int \dots d^4k$ and $\delta\left\{(k_\mu u^\mu)^2 - m^2\right\}$ are simple scalars, $\theta(k_\mu u^\mu)$ is a singular scalar, (θ is an odd function and k^μ and u^μ are simple and singular vectors, respectively), and $\bar{\Phi}(k)\Phi(k)$ is a singular pseudo-scalar, according to the definition (27) and eq. (26a). It is easy to construct a simple scalar

$$\int \left(\bar{\Phi} \Lambda_1 u^\mu \frac{\partial \Phi}{\partial x^\mu} - u^\mu \frac{\partial \bar{\Phi}}{\partial x^\mu} \Lambda_1 \Phi \right) \frac{dV}{\gamma}$$

which can, following [2,3], be interpreted as the particle mass while the nonlinear equation [4-7] is represented as follows:

$$i\lambda^\mu \frac{\partial \Phi}{\partial x^\mu} - \Phi \int \left(\bar{\Phi} \Lambda_1 u^\mu \frac{\partial \Phi}{\partial x^\mu} - u^\mu \frac{\partial \bar{\Phi}}{\partial x^\mu} \Lambda_1 \Phi \right) \frac{dV}{\gamma} = 0 \tag{30}$$

Unfortunately, the authors can only look at this fundamental (in our view) equation. It appears that any further progress in finding a solution to such an equation will be achieved with the help of computers and future symbol mathematics programs (of the Maple-16, Mathematica-9 types, etc.). For this purpose equation (30) should have a form with a clear matrix appearance. It is well known that the solution will not depend

on a concrete representation of matrices λ_μ, Λ_2 , it is only important that the commutations relations were satisfied. By the way, the latter can be checked by direct finding of commutators and anticommutators with apparent matrix representation. Let us note that the authors of [4-7] had received these results long before the epoch of personal computers and symbol math programs. When these things appeared, the first thing the authors did was to check the correctness of matrix correlations of the size 32x32!

In order to receive a concrete appearance of all the matrices, let us apply the bloc ideas. For this purpose, let us write down the basic matrices

$$\gamma_0, \gamma_1, \gamma_2, \gamma_3, g^{\mu\nu}, Z, i$$

$$\gamma_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad \gamma_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

$$\gamma_2 = \begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For these matrices the following standard commutation relations are correct:

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}; \quad \mu, \nu = 0, 1, 2, 3,$$

where $\mu, \nu, \sigma, \tau = 0, 1, 2, 3, 4$ and $g = +, -, -, -$.

From these basic matrices 10 supplementary bloc matrices can be constructed - $\lambda^{01}, \lambda^{02}, \lambda^{03}, \lambda^{04}, \lambda^{12}, \lambda^{13}, \lambda^{14}, \lambda^{23}, \lambda^{24}, \lambda^{34}$, which have a clear appearance:

$$\lambda_{01} = \begin{bmatrix} i & Z & Z & Z & Z & Z & Z & Z \\ Z & i & Z & Z & Z & Z & Z & Z \\ Z & Z & i & Z & Z & Z & Z & Z \\ Z & Z & Z & i & Z & Z & Z & Z \\ Z & Z & Z & Z & -i & Z & Z & Z \\ Z & Z & Z & Z & Z & -i & Z & Z \\ Z & Z & Z & Z & Z & Z & -i & Z \\ Z & Z & Z & Z & Z & Z & Z & -i \end{bmatrix}$$

$$\lambda_{02} = \begin{bmatrix} Z & Z & Z & Z & i & Z & Z & Z \\ Z & Z & Z & Z & Z & i & Z & Z \\ Z & Z & Z & Z & Z & Z & -i & Z \\ Z & Z & Z & Z & Z & Z & Z & -i \\ i & Z & Z & Z & Z & Z & Z & Z \\ Z & i & Z & Z & Z & Z & Z & Z \\ Z & Z & -i & Z & Z & Z & Z & Z \\ Z & Z & Z & -i & Z & Z & Z & Z \end{bmatrix}$$

$$\lambda_{23} = \begin{bmatrix} Z & Z & Z & Z & Z & Z & Z & -i \\ Z & Z & Z & Z & Z & Z & i & Z \\ Z & Z & Z & Z & Z & -i & Z & Z \\ Z & Z & Z & Z & i & Z & Z & Z \\ Z & Z & Z & -i & Z & Z & Z & Z \\ Z & Z & i & Z & Z & Z & Z & Z \\ Z & -i & Z & Z & Z & Z & Z & Z \\ i & Z & Z & Z & Z & Z & Z & Z \end{bmatrix}$$

$$\lambda_{03} = \begin{bmatrix} Z & Z & Z & Z & Z & Z & i & Z \\ Z & Z & Z & Z & Z & Z & Z & -i \\ Z & Z & Z & Z & i & Z & Z & Z \\ Z & Z & Z & Z & Z & -i & Z & Z \\ Z & Z & i & Z & Z & Z & Z & Z \\ Z & Z & Z & -i & Z & Z & Z & Z \\ i & Z & Z & Z & Z & Z & Z & Z \\ Z & -i & Z & Z & Z & Z & Z & Z \end{bmatrix}$$

$$\lambda_{24} = \begin{bmatrix} Z & Z & Z & Z & Z & Z & -i & Z \\ Z & Z & Z & Z & Z & Z & Z & -i \\ Z & Z & Z & Z & i & Z & Z & Z \\ Z & Z & Z & Z & Z & i & Z & Z \\ Z & Z & -i & Z & Z & Z & Z & Z \\ Z & Z & Z & -i & Z & Z & Z & Z \\ i & Z & Z & Z & Z & Z & Z & Z \\ Z & i & Z & Z & Z & Z & Z & Z \end{bmatrix}$$

$$\lambda_{04} = \begin{bmatrix} Z & Z & Z & Z & Z & Z & Z & \gamma_0 \\ Z & Z & Z & Z & Z & Z & \gamma_0 & Z \\ Z & Z & Z & Z & Z & \gamma_0 & Z & Z \\ Z & Z & Z & Z & \gamma_0 & Z & Z & Z \\ Z & Z & Z & \gamma_0 & Z & Z & Z & Z \\ Z & Z & \gamma_0 & Z & Z & Z & Z & Z \\ Z & \gamma_0 & Z & Z & Z & Z & Z & Z \\ \gamma_0 & Z & Z & Z & Z & Z & Z & Z \end{bmatrix}$$

$$\lambda_{34} = \begin{bmatrix} Z & Z & Z & Z & -i & Z & Z & Z \\ Z & Z & Z & Z & Z & -i & Z & Z \\ Z & Z & Z & Z & Z & Z & -i & Z \\ Z & Z & Z & Z & Z & Z & Z & -i \\ i & Z & Z & Z & Z & Z & Z & Z \\ Z & i & Z & Z & Z & Z & Z & Z \\ Z & Z & i & Z & Z & Z & Z & Z \\ Z & Z & Z & i & Z & Z & Z & Z \end{bmatrix}$$

$$\lambda_{12} = \begin{bmatrix} Z & Z & Z & Z & Z & Z & Z & \gamma_1 \\ Z & Z & Z & Z & Z & Z & \gamma_1 & Z \\ Z & Z & Z & Z & Z & \gamma_1 & Z & Z \\ Z & Z & Z & Z & \gamma_1 & Z & Z & Z \\ Z & Z & Z & \gamma_1 & Z & Z & Z & Z \\ Z & Z & \gamma_1 & Z & Z & Z & Z & Z \\ Z & \gamma_1 & Z & Z & Z & Z & Z & Z \\ \gamma_1 & Z & Z & Z & Z & Z & Z & Z \end{bmatrix}$$

$$\lambda_{13} = \begin{bmatrix} Z & Z & Z & Z & Z & Z & Z & \gamma_2 \\ Z & Z & Z & Z & Z & Z & \gamma_2 & Z \\ Z & Z & Z & Z & Z & \gamma_2 & Z & Z \\ Z & Z & Z & Z & \gamma_2 & Z & Z & Z \\ Z & Z & Z & \gamma_2 & Z & Z & Z & Z \\ Z & Z & \gamma_2 & Z & Z & Z & Z & Z \\ Z & \gamma_2 & Z & Z & Z & Z & Z & Z \\ \gamma_2 & Z & Z & Z & Z & Z & Z & Z \end{bmatrix}$$

$$\lambda_{14} = \begin{bmatrix} Z & Z & Z & Z & Z & Z & Z & \gamma_3 \\ Z & Z & Z & Z & Z & Z & \gamma_3 & Z \\ Z & Z & Z & Z & Z & \gamma_3 & Z & Z \\ Z & Z & Z & Z & \gamma_3 & Z & Z & Z \\ Z & Z & Z & \gamma_3 & Z & Z & Z & Z \\ Z & Z & \gamma_3 & Z & Z & Z & Z & Z \\ Z & \gamma_3 & Z & Z & Z & Z & Z & Z \\ \gamma_3 & Z & Z & Z & Z & Z & Z & Z \end{bmatrix}$$

$$u^\mu = (u_0, u_1, u_2, u_3) = \left(\frac{1}{\gamma}; \frac{\mathbf{v}}{\lambda}\right)$$

Let us define four-velocity

The matrices in the main equation (30) will be defined as:

$$\lambda^0 = 0 + \lambda^{01}u_1 + \lambda^{02}u_2 + \lambda^{03}u_3 + \lambda^{04}$$

$$\lambda^1 = \lambda^{01}u_0 + 0 + \lambda^{12}u_2 + \lambda^{13}u_3 + \lambda^{14}$$

$$\lambda^2 = \lambda^{02}u_0 + \lambda^{21}u_1 + 0 + \lambda^{23}u_3 + \lambda^{24}$$

$$\lambda^3 = \lambda^{03}u_0 + \lambda^{31}u_1 + \lambda^{32}u_2 + 0 + \lambda^{34}$$

The equation then will look as follows:

$$i \left(\lambda^0 \frac{\partial \Phi}{\partial x^0} + \lambda^1 \frac{\partial \Phi}{\partial x^1} + \lambda^2 \frac{\partial \Phi}{\partial x^2} + \lambda^3 \frac{\partial \Phi}{\partial x^3} \right) - m\Phi = 0 \tag{31}$$

The mass term of this equation will then be defined by the following correlation:

$$m = \int_V \left(\Phi^+ \Lambda_2 u^\mu \frac{\partial \Phi}{\partial x^\mu} - u^\mu \frac{\partial \Phi^+}{\partial x^\mu} \Lambda_2 \Phi \right) \frac{dV}{\gamma}$$

because $\bar{\Phi} = \Phi^+ \Lambda$; $\Lambda_1 = \lambda^{04} \lambda^{14} \lambda^{24} \lambda^{34}$;

$$\Lambda = \lambda^{12} \lambda^{13} \lambda^{14} \lambda^{23} \lambda^{24} \lambda^{34}; \Lambda_2 = \Lambda \Lambda_1$$

The explicit form of 4 matrices λ^μ depending on velocity, as well as of numerical matrices $\Lambda, \Lambda_1, \Lambda_2$ of the size 32x32. Using a good personal computer it is possible to prove the correctness of the correlations in (5) by making direct computations of the commutators and anticommutators with the help of symbol mathematics programs (Maple -16, Mathematica -9).

Solve equations

The attempts to solve equation of the (30), (31) type gave no result. However, [8-10] an interesting was found for a modified scalar version of the integro-differential equation (30), which may be written down as follows:

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\Phi(x, y, z, t) = -2i\Phi(x, y, z, t) * \tag{32}$$

$$* \int_0^x \int_0^y \int_0^z \Phi^*(x, y, z, t) \frac{\partial \Phi(x, y, z, t)}{\partial t} dx dy dz$$

We will seek the solution of this equation in the form $\Phi(x, y, z, t) = F(x, y, z) \exp(-i(\omega t - kx - ky - kz))$ where

$$F(x, y, z) = X(x)Y(y)Z(z)$$

and ω, k are some constant parameters. Substituting these expressions in (32), we obtain under condition $\omega = 3k$ following equation w.r.t X,Y,Z:

$$\frac{X'(x)}{X(x)} + \frac{Y'(y)}{Y(y)} + \frac{Z'(z)}{Z(z)} = -2\omega \int_0^x X^2(x) dx \cdot \int_0^y Y^2(y) dy \cdot \int_0^z Z^2(z) dz$$

Differentiating the left-hand and right-hand sides w.r.t x,y,z successively, we obtain three equations for $X(x), Y(y), Z(z)$:

$$\left(\frac{X'(x)}{X(x)}\right)' = -2\omega X^2(x) \int_0^y Y^2(y) dy \cdot \int_0^z Z^2(z) dz$$

$$\left(\frac{Y'(y)}{Y(y)}\right)' = -2\omega Y^2(y) \int_0^x X^2(x) dx \cdot \int_0^z Z^2(z) dz,$$

$$\left(\frac{Z'(z)}{Z(z)}\right)' = -2\omega Z^2(z) \int_0^x X^2(x) dx \cdot \int_0^y Y^2(y) dy.$$

Putting

$$U(x) = \int_0^x X^2(x) dx, \quad V(y) = \int_0^y Y^2(y) dy, \quad W(z) = \int_0^z Z^2(z) dz,$$

we obtain the system of ordinary differential equations for $X(x), Y(y), \dots, W(z)$:

$$X'' - \frac{(X')^2}{X} = -2\omega X^3 VW, U'(x) = X^2(x),$$

$$Y'' - \frac{(Y')^2}{Y} = -2\omega Y^3 UW, V'(y) = Y^2(y), \tag{34}$$

$$Z'' - \frac{(Z')^2}{Z} = -2\omega Z^3 UV, W'(z) = Z^2(z).$$

Further, we have put the numerical value of ω , namely, $\omega = \frac{1}{2}$ and integrated numerically (with the help of Maple-16) this system under following initial conditions (reasonable from physical point of view):

$X(0) = Y(0) = Z(0) = 1, X'(0) = Y'(0) = Z'(0) = U(0) = V(0) = W(0) = 0.$ According to obtained solution $X(x), Y(y), Z(z)$ are identical rapidly decreasing functions of following type:

$$X(x) \propto \exp(-x^p), \quad Y(y) \propto \exp(-y^p), \quad Z(z) \propto \exp(-z^p), \tag{35}$$

$1 < p < 2.$

The plot of $X(x)$ is shown in Fig.1.

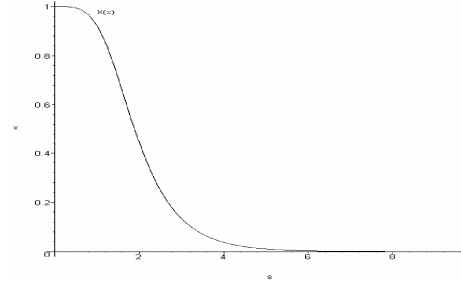


Fig.1

The basic equation (32) can be reduced to the scalar equation [5-7] for the density of the space charge of the space charge of the bunch, which represents the particles:

$$\frac{1}{c} \frac{\partial \Phi(r, t)}{\partial t} + \frac{\partial \Phi(r, t)}{\partial r} + \frac{4\pi i \Phi(r, t)}{h} * \int_0^r \left\{ \Phi^*(s, t) \frac{\partial \Phi(s, t)}{\partial t} - \frac{\partial \Phi^*(s, t)}{\partial t} \Phi(s, t) \right\} s^2 ds = 0 \tag{36}$$

Let us solve this equation together with the Poisson equation [5-7]:

$$\text{div grad } \varphi = -4\pi\rho$$

We seek the solution in the form

$$\Phi(r, t) = \bar{F}(r) \exp[-i(\omega t - kr)] \tag{37}$$

We get the following system of equations if the condition $\omega = kc$ is fulfilled:

$$\frac{d \bar{F}(r)}{dr} + \frac{8\pi\omega \bar{F}(r)}{h} \int_0^r s^2 \bar{F}^2(s) ds = 0,$$

$$\frac{d^2 \bar{\varphi}(r)}{dr^2} + \frac{2}{r} \frac{d \bar{\varphi}(r)}{dr} = -4\pi\rho(r) = -\frac{1}{2} \sqrt{\frac{c^3}{h}} \bar{F}^2(r), \tag{38}$$

where

$$\rho(r) = \frac{1}{8\pi} \sqrt{\frac{c^3}{h}} \bar{F}^2(r)$$

is the electrical charge density. Let us suppose

$$x = \frac{r}{R}, \quad f(x) = \frac{\bar{F}(r)}{\bar{F}(0)}, \quad \bar{F}(0) \neq 0$$

$$\rho(x) = \frac{2}{R^2 \bar{F}(0)} \sqrt{\frac{h}{c^3}} \bar{\varphi}(r), \quad K = \frac{8\pi\omega R^4 \bar{F}^2(0)}{h}$$

System (38) can be expressed in dimensionless form:

$$\frac{d^2 \ln f(x)}{dx^2} + Kx^2 f^2(x) = 0 \tag{39}$$

$$\frac{d^2 \rho(x)}{dx^2} + \frac{2}{x} \frac{d\rho(x)}{dx} = -f^2(x)$$

As long as potential ρ with the accuracy up to an additive constant and its value does not affect the intensity of electrical field $E = -grad\varphi$, let us choose $\varphi = 0$. Due to the spherical symmetry in the center of the particle, the condition $E = 0$ is fulfilled. Solving numerically the Cauchy problem for the system (39), taking the value $K = 16\pi$ and the initial conditions

$$f(0) = 1, f'(0) = 0, \varphi(0) = 0, \varphi'(0) = 0 \quad (40)$$

we obtain the following integrals

$$I_Q = \int_0^{\infty} x^2 f^2(x) dx = 8.51372561 \cdot 10^{-2}, I_Q^2 = 1/137.9623876 \quad (41)$$

$$I_E = \frac{1}{2} \int_0^{\infty} x^2 E^2(x) dx = 5.6857305 \cdot 10^{-3} \quad (42)$$

$$I_\mu = \int_0^{\infty} x^4 f^2(x) dx = 3.2493214 \cdot 10^{-2} \quad (43)$$

The quantity I_Q is a dimensionless electrical charge, which is brought to the following dimensional form:

$$Q = \sqrt{\hbar c} I_Q = 4.78709 \cdot 10^{-2} CGSE$$

This value is less than the modern experimental value of the electron's charge by only 0.3%. This is a fairly accurate number for the first theoretical attempt of the charge calculation. The plot of $f(x)$ is shown in Fig.1.

Thus it is not unusual to bring out the "corrections" of the J. Schwinger type to the integral (41)

$$I_e = I_Q + \frac{I_Q^2}{8\pi} - \frac{I_Q^3}{64\pi^2} = 8.5424692 \cdot 10^{-2}$$

which corresponds to the value of charge

$e = 4.803\ 2514 \cdot 10^{-10} CGSE$ and the value of fine-structure constant $\alpha = 1/137.03552$. Calculation spectrum masses all elementary particles see [10-12].

The quantization of the electrical charge and masses seems to be the consequence of the balance between the dispersion and nonlinearity, which determines stable solutions.

The found density distribution for the particle's electrical charge allows us to determine the electrical form factor for the same particle

$$F(q) = \int_V \rho(x) \exp[-iqx] dV \quad (44)$$

We regret that we have not succeeded in finding an analytical solution of eq. (39), but we are able to give a decent approximation. Let us look for a solution of eq. (39) in the form

$$f(x) = \operatorname{sech} R(x) \quad (45)$$

Substituting eq. (45) into eq. (39) and taking into account that for small R we have

$$\frac{1}{2} \sinh 2R \approx R$$

we obtain

$$(RR') = 16\pi x^2; R = \sqrt{\frac{8\pi}{3}} x^2 \quad (46)$$

$$f(x) = \operatorname{sech} \sqrt{\frac{8\pi}{3}} x^2 \quad (47)$$

It is interesting to note that if the particle's 4-velocity is assumed to be zero at matrix Λ , then system (30) will reduce to eight similar Dirac equations.

However, this requirement is absolutely unsatisfactory both from the physical and the mathematical points of view. Four-velocity has 4 components, of which three are usual components of the particle velocity along three axes, and they really can tend to zero. But the same cannot be done with the fourth component.

Problems

Hence, this approach is formally incorrect and requires explanation. In our view, although the Dirac equation describes the hydrogen atom spectrum absolutely correctly, it is not properly a fundamental equation. It has two weak points: the correct magnitude of the velocity operator's proper value is absent. It is known that in any problem of this type the proper value of the velocity operator is always equal to the velocity of light! In fact, Russian physicist and mathematician V.A.Fok regarded this as an essential defect of the Dirac theory.

The equations of the Unitary Quantum Theory we are proposing are more correct and fundamental. For this reason, a transition from correct fundamental equations to the incompletely accurate Dirac equation needs such a strange requirement as

$$u_\mu = 0$$

In the second paragraph of the preface of the book *A History of the Theories of Aether and Electricity*, by Sir Edmund T. Whittaker (Edinburgh, Scotland, April, 1951) was written the following:

"A word might be said about the title 'Aether and Electricity'. As everyone knows, the aether played a great part in the physics of the nineteenth century; but in the first decade of the twentieth, chiefly as a result of the failure of attempts to observe the Earth's motion relative to the aether, and the acceptance of the principle that such attempts must always fail, the word 'aether' fell out of favour, and it became customary to refer to the interplanetary spaces as 'vacuous'; the vacuum being conceived as mere emptiness, having no properties except that of propagating electromagnetic waves. But with the development of quantum electrodynamics, the vacuum has come to be regarded as the seat of 'zero-point' oscillations of the electromagnetic field, of the 'zero-point' fluctuations of electric charge and current, and of a 'polarization' corresponding to a dielectric constant different from unity. It seems absurd to retain the name 'vacuum' for an entity so rich in physical properties, and the historical word 'aether' may be fitly retained." Of course, now aether is not old aether of the nineteenth century.

The question is that the main relativistic relation between energy, impulse, and mass

$$E^2 = P^2 + m^2 \quad (48)$$

has been still beyond any doubt. In particular, all of the previous equations are based on relativistic invariance. Nevertheless, we shall ask ourselves once again about what is happening with that relation at the exact moment when the wave packet disappears being spread over the space. At that moment the particle does not exist as a local formation. This means that in the local sense there is no mass, local impulse, or energy. The

particle in that case, within sufficiently small period of time, is essentially non-existent, for it does not interact with anything. Perhaps this is why the relation (48) is average and its use at the wavelength level is equal or less than the De Broglie wavelength, which is just illegal. The direct experimental check of that relation at small distances and short intervals is hardly possible today. If the relation (48) is declined, then it may result in an additional conservation of energy and impulse refusal; but, as we know, according to the Standard Quantum Theory, that relation may be broken within the limits of uncertainty relation. On the other hand, the Lorentz's transformations have appeared when the transformation properties of Maxwell's equations were analyzing. However electromagnetic waves derived from solutions of Maxwell's equations move all in vacuum with the same velocity, i.e. are not subjected to dispersion and do not possess relativistic invariance. Our partial waves, which form wave packet identified with a particle, possess always the linear dispersion. Under such circumstances, it would be quite freely for authors to spread the requirement of relativistic invariance to partial waves. Such requirement has sense in respect only to wave packet's envelope, which appears if we observe a moving wave packet and his disappearance and reappearance. May be the origin of relativistic invariance would be connected in future with the fact that an envelope remains fixed in all inertial reference frames; only the wave's length is changed.

It's quite complicated [16,17]. The special relativity – is in fact Lorentz transformations (1904) derived by V.Vogt (1887) in the century before last. These transformations followed from the properties of Maxwell equations which are also proposed in the nineteenth century (1873). One of these equations connecting electrostatic field divergence and electric charge (Gauss' law of flux), in fact is just another mathematical notation of Coulomb's law for point charges.

But today anybody knows that Coulomb's law is valid for fixed point charges only. If charges are frequently moving Coulomb's law is not performed. Besides anybody knows that lasers beams are scattered in vacuum one over another, which is absolutely impossible in Maxwell equations. That means that Maxwell equations are approximate - and for the moving point charges experimental results essentially differs from the estimated ones in the case charges areas are overlapping.

Few people think about the shocking nonsense of presenting in any course of physics of point charge electric field in the form of a certain "sun" with field lines symmetrically coming from the point. But electric field – is a vector, and what for is it directed? The total sum of such vectors is null, isn't it?

There are no attempts to talk about, but such idealization is not correct. We should note that Sir Isaak Newton did not used term of a point charge at all, but it's ridiculous to think that such simple idea had not come to him! As for Einstein, he considered "electron is a stranger in electrodynamics". Maxwell equations are not ultimate truth and so we should forget, disavow the common statement about relativist invariance requirement being obligatory "permission" for any future theory.

To reassure severe critics we should note that UQT is relativistically invariant, it allows to obtain correct correlation between an energy and impulse, mass increases with a rate, as for relativistic invariance just follow of the fact that the envelope of moving packet is quiet in any (including non-inertial) reference systems. To be honest we should note that subwaves the particles consist of are relativistically abnormal, at the same

time envelope wave function following from their movement confirms terms of Lorentz transformations.

The success of Maxwell equations in description of the prior-quantum view of world was very impressing. Its correlation of the classical mechanics in forms of requirement to correspond Lorentz transformations was perfectly confirmed by the experiments that all these had resulted in unreasoned statement of Maxwell equations being an ultimate truth...

Other reasons for this were later very carefully investigated by a disciple of one of the authors (L.S.), Professor RatisYu.L. (S.Korolev Samara State Aero-Space University), who has formulated the modern spinor quantum electrodynamics from the UQT point of view:

1. Maxwell equations contain constant c , which is interpreted as phase velocity of a plane electromagnetic wave in the vacuum.
2. Michelson and Morley have never measured the dependence of the velocity of a plane electromagnetic wave in the vacuum on the reference system velocity as soon plane waves were mathematical abstraction and it was impossible to analyze their properties in the laboratory experiment in principle.
3. Electromagnetic waves cannot exist in vacuum by definition. A spatial domain where an electromagnetic wave is spreading – is no longer a vacuum. Once electromagnetic field arises in some spatial region at the same moment such domain acquires new characteristic – it became a material media. And such media possesses special material attributes including power and impulse.
4. Since electromagnetic wave while coming through the abstract vacuum (the mathematical vacuum) transforms it in a material media (physical vacuum) it will interact with this media.
5. The result of the electromagnetic wave and physical vacuum interaction are compact wave packets, called photons.
6. The group velocity of the wave packet (photon) spreading in the media with the normal dispersion is always less its phase velocity.

All abovementioned allows making unambiguous conclusion: the main difficulties of the modern relativistic quantum theory of the field arise from deeply fallacious presuppositions in its base. The reason for this tragic global error was a tripe substitution of ideas – velocity of electromagnetic wave packets ' c ' being transformed in numerous experiments physics have construed as constant ' c ' appearing in Maxwell equations and Lorentz transformations. Such blind admiration of Maxwell and Einstein geniuses (authors in no case do not doubt in the genius of these persons) had led XX century physics up a blind alley. The way out was in the necessity of revision of the entire fundamental postulates underlying the modern physics. Exactly that was done by UUQFT [14].

Some time ago CERN has conducted repeated experiments of the neutrino velocity measurement that appeared to be higher than velocity of the light. For UUQFT they were like a balm into the wounds. In fact the movements in excess of the light velocity were discovered earlier by numerous groups of researches. The most interesting were experiments of [19] (Princeton, USA), they had disclosed velocities 310 times higher than the light. Nearly everybody disbelieved it. And now the neutrino movements exceeding the velocity of the light were disclosed in CERN. The importance of these experiments for UUQFT is settled in the article [12-14] where at the page 69 it is written that "*this should be considered as direct experimental proof of UUQFT principle*".

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