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ON THE MASS SPECTRUM OF ELEMENTARY PARTICLES IN UNITARY QUANTUM THEORY

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Abstract

The particle is represented by the wave packet in nonlinear space-time continuum. Because of dispersion, the packet periodically appears and disappears in movement and the envelope of the process coincides with the wave function. There was considered the partial differential equation of telegraph-type describing the motion of such wave packet in spherical coordinate space(r, θ, φ). There was constructed also the analytical solution $u(r, \theta, \varphi)$ of this equation and the integral of $\left|grad|u|^2\right|^2$ over all space was supposed being equal to the

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mass of the particle identified with the wave packet.The solution $u(r, \theta, \varphi)$ depends on two positive integer parameter L, m and our theoretical particle's masses for different L, m were calculated. So, we have obtained the theoretical mass spectrum of elementary particles. The comparison with known experimental mass spectrum shows our calculated theoretical mass spectrum is sufficiently verisimilar.

In the standard quantum theory, a micro-particle is described with the help of a wave function with a probabilistic interpretation. This does not follow from the strict mathematical formalism of the non-relativistic quantum theory, but is simply postulated. A particle is represented as a point that is the source of a field, but can not be reduced to the field itself and nothing can be said about its "structure" except with these vague words.

There is a school in physics, going back to William Clifford, A. Einstein, and Louis de Broglie, where a particle is represented as a cluster or packet of waves in a certain unified field. According to M. Jemer's classification, this is a 'unitary' approach. The essence of this paradigm is clearly expressed by Albert Einstein's own words: " *We could regard substance as those areas of space where a field is immense. From this point of view, a thrown stone is an area of immense field intensity moving at the stone's speed. In such new physics there would be no place for substance and field, since field would be the only reality... and the laws of movement would automatically ensue from the laws of field."*

The Unitary Quantum Theory (UQT) represents a particle as a bunched field (cluster) or a packet of partial waves with linear dispersion [1-11]. Dispersion is chosen in such a way that the wave packet would periodically disappear and appear in movement, and the envelope of the process would coincide with the wave function. Based on this idea, the relativistic-invariant model of such unitary quantum field theory was built. The principal nonlinear relativistic invariant equation is following [6,10,11]:

$$i\lambda^{\mu}\frac{\partial\Phi}{\partial x^{\mu}} - \frac{c\Phi}{\hbar}\int\left(\bar{\Phi}\lambda_1 u^{\mu}\frac{\partial\Phi}{\partial x^{\mu}} - u^{\mu}\frac{\partial\bar{\Phi}}{\partial x^{\mu}}\lambda_1\Phi\right)\frac{dV}{\gamma} = 0, \qquad (1)$$

where $x^{\mu} = (ct, x)$, $\gamma = \sqrt{1 - v^2/c^2}$, v is the velocity, $u^{\mu} = \left(\frac{1}{\gamma}, \frac{v}{\gamma}\right)$ is the four-velocity of a particle, matrices $\lambda^{\mu}(32 \times 32)$ satisfy the commutation relations

$$\lambda^{\mu}\lambda^{\nu} + \lambda^{\nu}\lambda^{\mu} = 2g^{\mu\nu}I, \mu, \nu = 0, 1, 2, 3,$$

$g^{\mu\nu}$ is the metrical tensor, I is the unity matrix and λ_1 is the product of four corresponding matrices λ^{μ}. This fundamental equation of UQT describes, in our opinion, all properties of elementary particles. It is possible to derive from(1) the Dirac equation and also the relativistic invariant Hamilton - Jacoby equation [2,3]. We have succeeded in solving only the simplified scalar variant of eq.(1).However, the solution obtained has allowed to determine theoretically the elementary electrical charge and the fine-structure constant with high precision (our theoretical value $\alpha = 1/137.962$ and the known experimental value $\alpha = 1/137.03552$). Our efforts to find more complete solution of eq.(1) were unsuccessful.

Nevertheless, our idea to consider a particle as some moving wave packet which periodically disappears and appears in movement, has allowed to arrive to the conclusion [9-11] that such particle may be described by the common telegraph - type equation of the second order. In one- dimension case this equation is following:

$$\frac{1}{v^2}\frac{\partial^2 F(x,t)}{\partial t^2} - \frac{\partial^2 F(x,t)}{\partial x^2} - \frac{2imc^2\sqrt{1-v^2/c^2}}{\hbar v^2}\frac{\partial F(x,t)}{\partial t} + \\ - \frac{m^2c^4(1-v^2/c^2)}{\hbar^2 v^2}F(x,t) = 0. \quad (2)$$

(Note, this equation would be relativistic invariant if the root $\sqrt{1-v^2/c^2}$ would be placed in denominator.)

Equation (2) is satisfied exactly by relativistic invariant solutions in the form of a standard planar quantum-mechanical wave and also in the form of disappearing and appearing wave-packet, viz.,

$$F(x,t) = \exp\left(\frac{i}{\hbar}\frac{mc^2t - mvx}{\sqrt{1-v^2/c^2}}\right) \quad (3)$$

or

$$F(x,t) = \exp\left(\frac{i}{\hbar}\frac{mc^2t - mvx}{\sqrt{1-v^2/c^2}}\right)\varphi(x-vt), \quad (4)$$

where φ is an arbitrary function of its argument $x - vt$.

We will show that eq. (2) (considered in the case of 3-dimension coordinate space (r, θ, φ)) allows to determine theoretically the mass spectrum of elementary particles.

Such equation for the function$u = u(r,\theta,\varphi,t)$ is following:

$$\frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} - \frac{1}{r^2\sin\theta}\left(2r\sin\theta\frac{\partial u}{\partial r} + r^2\sin\theta\frac{\partial^2 u}{\partial r^2} + \cos\theta\frac{\partial u}{\partial \theta} + \right. \\ \left. + \sin\theta\frac{\partial^2 u}{\partial\theta^2}\,\frac{1}{\sin\theta}\frac{\partial^2 u}{\partial\varphi^2}\right) - \frac{2iMc^2\sqrt{1-v^2/c^2}}{v^2\hbar}\frac{\partial u}{\partial t} \\ - \frac{M^2c^4(1-v^2/c^2)}{v^2\hbar^2}u = 0, \quad (5)$$

(the symbol m is replaced by M).

We will use the natural system of units and put $\hbar = 1, c = 1$ and will seek the solution of eq. (5) in following form:

$$u = \frac{f}{r}\exp\left(\frac{iMt}{\sqrt{1-v^2}} - \frac{iMvr}{\sqrt{1-v^2}}\right), \quad (6)$$

where $f = f(r,\theta,\varphi)$ is some function not depending on t. This function represents as if hardened wave packet in coordinate space (r,θ,φ). Substituting (6) in equation(5) , we get

$$-2iMvr^2\sin^2\theta\frac{\partial f}{\partial r} + \sqrt{1-v^2}\left(r^2\sin^2\theta\frac{\partial^2 f}{\partial r^2} + \sin^2\theta\frac{\partial^2 f}{\partial\theta^2} + \right. \\ \left. \frac{\partial^2 f}{\partial\varphi^2} + \sin\theta\cos\theta\frac{\partial f}{\partial\theta}\right) = 0. \quad (7)$$

We will seek the solution of eq. (7) in form:

$$f = R(r)Y_{Lm}(\theta,\varphi), \quad (8)$$

where

$$Y_{Lm}(\theta,\varphi) = \frac{\sqrt{(2L+1)(L-m)!}}{2\sqrt{\pi(L+m)!}}P_L^m(\cos\theta)\exp(\pm im\varphi), \quad (8‘)$$

$P_L^m(\cos\theta)$ is the Legendre function,$Y_{Lm}(\theta,\varphi)$ is the spherical harmonic and L, m are nonnegative integers $L = 0,1,2,3,\ldots,$ $m = 0,1,2,3,\ldots,$ $m \le L$. Substituting (8) in eq. (7), we come to the following equation for$R(r)$:

$$\frac{d^2R(r)}{dr^2}r^2\sqrt{1-v^2} - 2i\frac{dR(r)}{dr}Mvr^2 - \sqrt{1-v^2}R(r)L(L+1) = 0. \quad (9)$$

The solution of this equation $R_L(r)$ depends on parameter L and we obtain the family of solutions $u_{Lm}(r, \theta, \varphi, t)$ of eq. (9) depending on parameters L, m.

It is natural to suppose that every solution u_{Lm} of our equation (5) describes the amplitude of a partial world unitary potential $\Phi_{L,m}$ determined by partial wave packet and the potential itself is represented by the quadrate of amplitude modulus, i.e.

$$\Phi_{Lm} = |u_{Lm}|^2 = \left| \frac{R_L(r)}{r} Y_{Lm}(\theta, \varphi) \right|^2 . \tag{10}$$

Further, we consider the gradient of this potential as the tension of corresponding field (it is the custom in electrodynamics) of the partial wave packet and consider the quadrate of the tension as the density W_{Lm} of energy or of the wave packet‘s mass distributed continuously in space.If we consider eq. (9) in some fixed spherical zone Q_r of radius r , where the corresponding part of our hardened wave packet is placed, then it is natural to consider $M = M_{Lm}$ as the mass of this part of the wave packet, i.e. as the integral of density W_{Lm} over given spherical zone. Such approach allows to replace the mass M in eq. (9) by integral

$$M_{Lm} = \iiint\limits_{Q_r} W_{Lm} r^2 \sin(\theta) dr d\theta d\varphi, \tag{11}$$

where $W_{Lm} = |grad \Phi_{Lm}|^2$. So, we will consider eq.(9) as the integro-differential equation for $R_L(r)$. For the sake of simplicity, we will use the following expression for M_{Lm} (after discarding the members which depend on θ, φ) :

$$M_{Lm} = \int_0^r \left| \frac{d}{dr} \frac{R_L(r)^2}{r^2} \right|^2 r^2 dr. \tag{12}$$

We will use the following way to solve our integro-differential eq. (9). Viz., at first, we rewrite this equation in form

$$2ivM_{Lm} = \frac{1}{r^2 R'(r)} (R''(r) r^2 - L(L+1) R(r)) \sqrt{1 - v^2}, (' = \frac{d}{dr}). \tag{13}$$

(Here and further,we write the function $R(r)$ without subindex L.)

Secondly, we substitute the integral (12) for M_{Lm} in (9) and differentiate left and right-hand with respect to r. We obtain the equation

$$2iv\left[\frac{d}{dr}\left(\frac{R^2}{r^2}\right)\right]^2 r^2 = \frac{d}{dr}\left[\frac{1}{r^2R'}(R''r^2 - L(L+1)R)\right]\sqrt{1-v^2}. \tag{13'}$$

At the third step, we set $v = 0$ in (13'). The grounds are following The solution of this equation depends on parameter v (the velocity of our particle). It is natural to suppose that the potentials Φ_{Lm} describe processes which are continuous with respect to v (in any case, if v is less, than light velocity c),i.e. $\lim R(r, v) = R(r, v*)$,if $v \to v*$ and it is valid if $v* = 0$. Besides,we want to determine the inner (proper) characteristic of our wave packet not depending on the velocity of its movement. So, we set $v = 0$ and obtain more simple (although sufficiently complicate) non-linear differential equation for $R(r)$ (after corresponding differentiation):

$$\frac{d^3R(r)}{dr^3}\frac{dR(r)}{dr}r^3 + \frac{d^2R(r)}{dr^2}r\left(R(r)L(L+1) - \frac{d^2R(r)}{dr^2}r^2\right) + \\ - L(L+1)r\left(\frac{dR(r)}{dr}\right)^2 + 2L(L+1)R(r)\frac{dR(r)}{dr} = 0. \tag{14}$$

Fortunately, this equation possess the analytical general solution (in addition to trivial constant solution):

$$R(r) = C_2 \exp\left(-\frac{C_1}{2}r\right)\sqrt{r}J\left(L+\frac{1}{2}, \frac{1}{2}\sqrt{-C_1^2 r}\right) + \\ C_3 \exp\left(-\frac{C_1}{2}r\right)\sqrt{r}Y\left(L+\frac{1}{2}, \frac{1}{2}\sqrt{-C_1^2 r}\right), \tag{15}$$

where C_1, C_2, C_3 are arbitrary constants and J and Y are the Bessel functions. Since we seek the finite solution $R(r)$ if $r \to 0$ and tending to zero if $r \to \infty$, we set $C_3 = 0$ and can set some positive value for C_1, C_2. The calculations show the choice of these constants has influence only on the absolute value of the masses calculated below but the ratios of these masses remain the same. We have chosen the simplest values

$$C_1 = 2, C_2 = 1, C_3 = 0$$

and have obtained following solution

$$R(r) = \sqrt{r}\exp(-r)J(L+1/2, ir), \tag{16}$$

where $J(L+1/2, ir)$ is the Bessel function of the 1st type with imaginary argument, or

$$R(r) = i^{L+1/2}\sqrt{r}exp(-r)I(L+1/2, r), \tag{16'}$$

where $I(L+1/2, r)$ is the modified Bessel function of the 1st type.

So, we obtain the following expression for Φ_{Lm} (taking into consideration (6,8,8',10)):

$$\Phi_{Lm} = \frac{\exp(-2r)}{4\pi r}\left|\frac{(2L+1)(L-m)!(I(L+1/2,r))^2(P_L^m(\cos\theta))^2}{(L+m)!}\right| \tag{17}$$

Now, we form $grad\Phi_{Lm}$ considered as the tension of the world unitary field and form also the quadrate of its modulus considered as the mass density W_{Lm} of the corresponding partial wave packet. We obtain

$$W_{Lm} = \frac{\exp(-4r)(L-m)!^2(L+1/2)^2}{(L+m)!^2\pi^2 r^4}(I(L+1/2,r)^2F1+ \\ I(L+1/2,r)^4F2/\sin^2\theta), \tag{18}$$

where

$$F1 = \left[(L+r+1)I(L+1/2,r) - rI(L-1/2,r)\right]^2 P_L^m(\cos\theta)^4,$$

$$F2 = \left[((m-L-1)P_L^m(\cos\theta)P_{L+1}^m(\cos\theta) + (L+1)\cos\theta P_L^m(\cos\theta)^2\right]^2 \tag{18'}$$

The integrals of W_{Lm} over all spherical space (r, θ, φ) for different $L = 0, 1, 2, \ldots$ and $m = 0, 1, 2, \ldots, m \leq L$ are equal to required different masses M_{Lm} of elementary particles, i.e.

$$M_{Lm} = \int_0^\infty\int_0^\pi\int_0^{2\pi} W_{Lm} r^2 \sin\theta dr d\theta d\varphi \tag{19}$$

Since W_{Lm} does not depend on φ and the Legendre functions $P_L^m(\cos\theta)$ are integrable in analytical form with respect to θ, we derived, at first,(with the help of *Mathematica 5*) analytical expressions for the integrals

$$\int_0^{\pi}\int_0^{2\pi} W_{Lm} r^2 \sin(\theta) d\theta d\varphi = 2\pi \int_0^{\pi} W_{Lm} r^2 \sin(\theta) d\theta \tag{20}$$

denoting these integrals by U_{Lm}. Further, we calculated numerically the integrals

$$M_{Lm} = \int_0^{\infty} U_{Lm} dr \tag{21}$$

For example, we have obtained in the case of $L = 0, m = 0$

$$U_{00} = \frac{8\exp(-4r)\sinh^2(r)}{\pi^3 r^4}\left\{(r^2 + 1/2 + r)\cosh^2(r) + \right. \\ \left. -r(1+r)\sinh(r)\cosh(r) - (1+r)^2/2\right\}$$

and

$$M_{00} = \int_0^{\infty} U_{00} dr = 0.003944364169$$

In the case of $L = 1, m = 1$, we have obtained

$$U_{11} = \frac{8\exp(-4r)}{\pi^3 r^8} F0(r), \tag{22}$$

where

$$F0 = \left(r^6 + 5r^5 + \frac{93}{8}r^4 + 13r^3 + \frac{61}{4}r^2 + 2r + \frac{17}{8}\right)\cosh^4(r) - \\ - r\sinh(r)\cosh^3(r)\left(r^5 + 5r^4 + 11r^3 + \frac{33}{2}r^2 + 8r + \frac{17}{2}\right) - \\ - \cosh^2(r)\left(\frac{1}{2}r^6 + 3r^5 + 10r^4 + 14r^3 + \frac{71}{4}r^2 + 4r + \frac{17}{4}\right) + \\ r\sinh(r)\cosh(r)\left(r^4 + 3r^3 + 8r^2 + 8r + \frac{17}{2}\right) + \frac{1}{2}r^4 + r^3 + \frac{5}{2}r^2 + 2r + \frac{17}{8},$$

and

$$M_{11} = 0.00006798678730.$$

In the cases of small L, the calculations are sufficiently simple. But in the cases of large L, the expressions for U_{Lm} are represented by very long polynomials in r,cosh(r),sinh(r) with enormous numerical coefficients.The integration of these polynomials was realized only with *Mathematica5*.

We consider the ensemble of $L+1$ particles (masses) with given L and $m = 0, \ldots, \pm L$ to be one family and we will use the notations $M_{L,0}, M_{L,1}, \ldots, M_{L,L}$ for particles (masses) of the family with given L. We have calculated and analyzed in full the masses of 49 families ($L = 0, \ldots, 48$), i.e. of 1225 particles. Our PC with $3GHz, RAM = 4GB$ has required for these calculations nearly 3 weeks of computing time. Note, for negative m, we have obtained the same values of masses (antiparticles ?)

We have compared our theoretical spectrum for 1225 masses with known experimental spectrum for elementary particles measured in MeV. The zero-point for the matching of both spectra was required. For such matching, we have taken the quotient of the muon mass to the electron mass. As we know, this quotient for observed muons and electrons is measured experimentally with the most precision and is accepted to be equal to 206.76884. Each our calculated mass was divided consecutively by all other 1224 masses and the resulting quotients were compared with the mentioned value. It turned out that the quotient of our masses $M_{16,10}$ and $M_{48,45}$ is equal to 206.7607796 (with relative divergence 0.0039%) and we have taken our mass $M_{48,45}$ equal to 0.2894982442536304E-10 for zero-point, i.e. for our electron mass. Then, all other 1224 masses $M_{L,m}$ were divided by $M_{48,45}$ and we have obtained our theoretical spectrum in electron masses which may be compared (after expressing in MeV) with known experimental masses. Below, we give the table with our masses M_{Lm} for 30 cases of the well coincidence with well known experimental values (relative errors are less than 1% in 27 cases and between 1.3% and 1.8% in three cases).

M_{Lm}	*Theory*	*Experiment*	*Notation*	*Error%*
$M_{48,45}$	0.51099906	0.51099906	e	_
$M_{16,10}$	105.6545640	105.658387	μ	0.0036
$M_{18,4}$	135.8958708	134.9739	π^0	0.683
$M_{23,0}$	137.2902541	139.5675	π^+, π^-	1.62
$M_{14,1}$	541.7587460	548.86	η	1.29
$M_{7,7}$	894.0806293	891.8	$K^{\star+}, K^{\star 0}$	0.25
$M_{12,1}$	936.3325942	938.2723	p	0.206
$M_{10,4}$	957.1290490	957.2	ω	0.0083
$M_{9,5}$	1110.473414	1115.63	Λ	0.462
$M_{8,6}$	1224.151552	1233	b_1^0	0.71
$M_{11,1}$	1271.916682	1270	$K^\star$	0.14
$M_{9,4}$	1331.705434	1321.32	Ξ^-	0.78
$M_{10,2}$	1378.127355	1382.8	Σ^0	0.33
$M_{12,0}$	1524.617683	1520.1	Λ_2	0.29
$M_{8,5}$	1549.444919	1540 ± 5	F_1	0.28
$M_{7,6}$	1595.510637	1594	ω_1	0.094
$M_{9,3}$	1601.282953	1600	ρ'	0.08
$M_{6,6}$	1718.917400	1720	N_0^3	0.06
$M_{10.1}$	1774.917815	1774	$K_3^{\star+}$	0.051
$M_{8,4}$	1906.842877	1905	Δ_5^+	0.096
$M_{9,2}$	1965.115639	1950	Δ_4	0.77
$M_{11,0}$	2092.497779	2100	Λ_4	0.35
$M_{7,5}$	2195.695293	2190	$N(2190)$	0.25
$M_{7,4}$	2818.645188	2820	η_c	0.048
$M_{6,5}$	3082.979571	3096	J/ψ	0.42
$M_{7,3}$	3543.664516	3556.3	χ	0.35
$M_{5,5}$	3687.679612	3686.0	ψ'	0.04
$M_{7,2}$	4496.650298	4415	ψ'''	1.84
$M_{5,3}$	9499.927309	9460.32	Υ'	0.41
$M_{6,1}$	10075.78271	10023.3	Υ''	0.523
$M_{7,0}$	10533.15222	10580	Υ'''	0.442
$M_{0,0}$	6962274	?	*Dzhan*	?

(e-electron, μ-muon, π^0-meson, p - proton etc.)

Note, the ratio of our proton mass $M_{12,1}$ and our electron mass $M_{48,45}$ is equal 1832.355 with relative error 0.207% in comparison with well known experimental ratio 1836.152167. Our calculated spectrum containing 169 masses from muon to the heaviest mass approximates also others well known particles and the coincidences with experimental data are worse but quite acceptable (with relative divergences not more than several per cent).

On the whole, this table shows the striking coincidences of our theoretical values with essential quantity of the known experimental masses and such coincidence may be called,by no means, occasional. Note, the choice of the nominee for the electron's mass is not unique and may be further calculations of families with $L = 49 \ldots 100$ would allow to obtain the better result.

We have carried out also the series of calculation M_{Lm} for L exceeding 48 including $L = 60$. The ratio of maximal $M_{0,0} = 0.0039443641689$ to minimal $M_{60,60} = 0.3909395521 \cdot 10^{-11}$ is of order 10^9 . The ratio of maximal $M_{0,0}$ to the mass $M_{12,1} = 0.53046407119 \cdot 10^{-7}$ of proton is equal 74400.These numbers do not contradict the known experimental data.

Note, all radial functions $U_{Lm}(r)$ being the density mass as function of r, are equal zero for $r = 0$ and, at first, increase very swiftly on the right from for $r = 0$ and then very swiftly decrease. For large L, the plot of U_{Lm} reminds a delta-function $\delta(+0)$.Such theoretical model describes a particle as very small bubble in space-time continuum cut by spherical harmonics. Curious, such model was considered by A. Poincare [12].

Certainly, we do not intend to assert that our results are adequate in full to the known experimental mass spectrum of elementary particles. The divergences are present. Our theoretical spectrum contains the large quantity (1053) of masses between the electron mass and the muon mass but such real particles are not observed till now. Our spectrum contains many light particles with masses differing extremely little one from another. It may be supposed there exists quasi-continuous distribution of lightest particles not affirmed till now by experiments. Our spectrum contains 169 particles from the muon to the heaviest particle but there is observed the large quantity of particles in this interval with short "life-time" (so called "resonances") of order $10^{-20}sec.$ These divergences require the further researches.

With respect to light particles, it may be supposed there exist some selection principles (not discovered till now theoretically) for such particles and these principles lead to essential decreasing of particles quantity with masses between muon's and electron's masses. We suppose that such principles arise theoretically from some relations between the tensors of different valences (ranks) and spherical functions for different L, m. It arise also the question concerning particles with short "life-time": may we take all these particles for elementary? Our Unitary Quantum Theory allows to formulate the following criterion. " If the way which one particle (which we identify with appearing and disappearing wave packet) passes from the moment of its appearing to the moment of its destruction is much longer than de Broglie wave, then such particle may be called elementary". Have we reason to call "elementary" the particle with life-time of order $10^{-20} sec$?

Let us point to following essential circumstance. Viz., if we will use the Schroedinger equation in spherical coordinates (relativistic-non-invariant) or Klein-Gordon equation (relativistic-invariant) instead our initial equation (5), then we will come to the same theoretical mass spectrum. Really, the mention Schroedinger equation is following:

$$\frac{\hbar^2}{2Mr^2\sin\theta}\left(2r\sin\theta\frac{\partial u}{\partial r}+r^2\sin\theta\frac{\partial^2 u}{\partial r^2}+\cos\theta\frac{\partial u}{\partial\theta}+\sin\theta\frac{\partial^2 u}{\partial\theta^2}+\frac{1}{\sin\theta}\frac{\partial^2 u}{\partial\varphi^2}\right)+i\hbar\frac{\partial u}{\partial t}=0, \quad (22)$$

where M is the particle's mass. We will seek the solution of this equation in form of unitary wave packet :

$$u=\frac{f}{r}\exp\left(-i\frac{Mv^2}{2\hbar}t+i\frac{Mv}{\hbar}r\right), \quad (23)$$

where $f = f(r,\theta,\varphi)$ is the function of coordinates and does not depend explicitly on the time. Substituting (23) in (22), we obtain (after simplification) following equation:

$$\hbar r^2\sin^2\theta\frac{\partial^2 f}{\partial r^2}-2iMvr^2\sin^2\theta\frac{\partial f}{\partial r}+\frac{\hbar}{2}\sin 2\theta\frac{\partial f}{\partial\theta}+\hbar\sin^2\theta\frac{\partial^2 f}{\partial\theta^2}+\hbar\frac{\partial^2 f}{\partial\varphi^2}=0. \quad (24)$$

If we put $\sqrt{1-v^2}$ instead of $\hbar$,then this equation coincides with our equation (7). The further analysis remains without any changes.

Let us consider the Klein-Gordon equation in spherical coordinates and in natural unit system ($c = 1, \hbar = 1$):

$$\frac{1}{r^2\sin\theta}\left(2r\sin\theta\frac{\partial u}{\partial r} + r^2\sin\theta\frac{\partial^2 u}{\partial r^2} + \cos\theta\frac{\partial u}{\partial\theta} + \sin\theta\frac{\partial^2 u}{\partial\theta^2} + \frac{1}{\sin\theta}\frac{\partial^2 u}{\partial\varphi^2}\right) - \frac{\partial^2 u}{\partial t^2} - M^2 u = 0, \quad (25)$$

where M is the particle‘s mass. We will seek the solution in form:

$$u = \frac{f}{r}\exp\left(\frac{iMt}{\sqrt{1-v^2}} - \frac{iMvr}{\sqrt{1-v^2}}\right), \quad (26)$$

where $f = f(r, \theta, \varphi)$ does not depend explicitly on t. Substituting (26) in (25), we obtain following equation

$$\sqrt{1-v^2}\left(r^2\sin^2\theta\frac{\partial^2 f}{\partial r^2} + \sin^2\theta\frac{\partial^2 f}{\partial\theta^2} + \frac{\partial^2 f}{\partial\varphi^2} + \frac{1}{2}\sin 2\theta\frac{\partial f}{\partial\theta}\right) + - 2ivr^2 M\sin^2\theta\frac{\partial f}{\partial r} = 0. \quad (27)$$

This equation coincides in full with our equation (7) and we will come to the same results. So, different initial equation (5),(22),(25) (the last is relativistic invariant and the other two are relativistic non-invariant) lead to the same theoretical mass spectrum.

Below, the table with all our theoretical masses from the muon to the heaviest $M_{0,0}$ is given.

In view of all said above, we are bold, nevertheless, to say that our results represent the substantial advancement on the way of solution for the extremely complicated theoretical problem of the mass spectrum for elementary particles and to underline that this advancement is owing to our Unitary Quantum Theory.

We would like to propose the name ”Dzhan-particle” for our heaviest particle $M_{0,0}$. As we know, particles with mass of such order are observed in cosmic rays.

105.655	105.94	106.241	108.291	108.997	109.597	110.133	112.784
117.054	118.136	120.31	121.826	122.664	125.522	125.71	127.187
127.237	127.306	131.445	133.013	135.896	137.29	142.287	144.326
145.96	147.309	147.698	149.62	149.905	153.765	153.827	159.796
162.135	162.192	165.33	172.249	177.091	178.559	178.758	180.585
180.895	187.69	192.661	192.917	195.832	199.852	203.297	205.588
209.097	218.681	219.639	221.135	224.061	225.089	231.432	231.656
241.805	249.092	252.972	253.184	269.993	270.91	276.443	280.151
281.016	289.488	300.299	301.848	304.024	314.364	318.997	335.848
339.955	341.136	342.52	349.235	357.381	366.838	373.402	402.126
408.316	423.36	423.429	432.83	445.413	459.388	461.593	472.253
504.945	521.772	529.951	531.566	539.326	541.759	560.236	571.51
606.559	619.012	672.537	686.757	705.247	705.477	730.141	738.98
812.354	828.374	866.997	894.081	897.982	915.038	936.333	957.129
996.316	1110.47	1135.57	1137.9	1224.15	1271.92	1331.71	1378.13
1524.62	1549.43	1595.51	1601.28	1718.92	1774.92	1906.84	1965.1
2092.5	2195.7	2334.9	2557.69	2818.65	2906.6	2954.55	3082.98
3543.66	3687.68	3832.21	4300.87	4315.87	4496.65	5642.23	6026.01
6570.85	6666.64	7358.75	9219.36	9499.93	10075.8	10533.2	12941.1
16897	18035.6	18261.3	25000.7	28935.4	33698.9	36955.4	54518.8
71060.4	87704.5	131517	179100	266419	601983	$1.20005 \cdot 10^6$	$3.4545 \cdot 10^6$
$6.96227 \cdot 10^7$							

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Comment

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Comments on ON THE MASS SPECTRUM OF ELEMENTARY PARTICLES IN UNITARY QUANTUM THEORY

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Contemporary physics of elementary particles is dominated by field-theoretical ideas used to describe the "propagation" of point-like particles in spacetime. Aside of its many successes, this approach introduces problems of its own. Their origin is often attributed to the underlying notion of point. The idea that macroscopic space forms a continuum analogous to that of the reals, and thus that space may be divided again and again without ever encountering any conceptual change, constitutes a bold extrapolation from our everyday experience. Consequently, it may and should be questioned. In other words, both the concept of point underlying the field-theoretical formalism, and the concept of point-like particle may be approximations or mathematical idealizations only.

The problem of mass constitutes one of the essentially untouched problems of elementary particle physics. The present paradigm is that particle masses are generated by the Higgs mechanism. However, despite the important rôle of the latter in endowing the weak interaction gauge bosons with mass while at the same time ensuring the renormalizability of the theory, the Higgs mechanism does not really help us with the problem itself - it just hides it in an eye-catching clothing.

The stated goal of the authors is to get rid of the description in terms of both point-like particles and fields, and to represent all particles as objects extended over those regions of space where fields are very large (thus disposing of one point-like aspect of standard theory). Following this idea, the authors set up their generic equation for the field. If their approach is meaningful, its solutions should yield the allowed field energies, and - when the velocity of the described particles is set to zero - these energies should be equal to particle masses. After solving the equation, the authors compare a subset of its solutions with a subset of the experimental spectrum of elementary particles, and find that some of the obtained mass ratios are close to some experimental mass ratios.

In order to judge this result one needs a method to resolve the question whether such "successful predictions" are meaningful or just accidental. The important point is the number of parameters used, the definition of what is meant by success, the number of successes vs. the number of failures, the scope of problems discussed and their interrelations, etc...

Judging in this way, the alleged "success" is a failure.

First, the paper predicts a huge number of unseen light particles and lacks an acceptable explanation of their absence in the experimental mass spectrum. Thus, the number of failures is huge.

Second, the proposed approach completely ignores the fact that the experimentally observed particles may be gathered together into different groups according to various "internal" quantum numbers (the simplest being the lepton and baryon numbers) and that their masses depend on these quantum numbers in an essential way. Accordingly, the observed mass spectrum of elementary particles shows various important regularities. In addition, everything indicates that the spectrum of hadrons is unbounded from above. At least a part of the issue of internal quantum numbers *must* be taken into account in any sensible approach to particle masses.

Third, the problem of mass is further complicated by the problem of quark masses - again ignored by the authors completely. One has to consider not only the existence of quarks and the assignement of masses to them, but also the fact that contemporary approaches introduce two types of quark mass: the "current" mass and the "constituent" mass. One has to take into account that - in the standard description - quarks and leptons interact as if they were ordinary point-like particles, and yet they are obviously different at a fundamental level as far as their masses are concerned. Indeed: the mass of an electron, or proton is easily measurable, while present "measurements" of quark masses are loaded with theoretical inputs and conceptual inconsistencies. An approach to the problem of mass that may be considered as conceptually at least partially satisfactory has to get rid of at least some of such inconsistencies. These problems were extensively discussed e.g. in [1], where phase-space-related origin of some of the observed quantum numbers and their possible connection with the concept of mass have been suggested.

Any serious attempt to deal with the problem of mass must consider at least some of such questions. This paper, besides various other problems, does not even admit their existence, however.

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Authors' response

In existing quantum physics all elementary particles are described by points which are the sources of a field, but not reduced to the field itself. This dualism is absolutely not satisfactory as the two substances have been introduced, that is, both the points and the fields. The points, that is the sources of a field, but not driven to the field. Presence of both points and fields at the same time is not satisfactory from general philosophical positions - razors of Ockama. Besides that, the presence of the points leads to non-convergences, which are eliminated by various methods, including the introduction of a renormalization group that is declined by many mathematicians and physicists, for example, P.A.M. Dirac. Modern quantum field theory can not even formulate the problem of finding a mass spectrum. The original idea of Schroedinger was to represent a particle as a wave packet of de Broglie waves. As he wrote in one of his letters, he "was happy for three months" before British mathematician Darwin showed that the packet quickly and steadily dissipates and disappears. Then it turns out that this beautiful and unique idea to represent a particle as a portion of a field is not realizable in the context of wave packets of de Broglie waves. Later, de Broglie tried to save this idea by introducing nonlinearity for the rest of his life, but wasn't able to obtain significant results. Yet later, the idea of Einstein's unitary program had appeared. However, its realization appeared to be possible only in the context of the Unitary Quantum Theory (UQT) within last two decades. It is impressive, that the problem of mass spectrum has been reduced to exact analytical solution of a nonlinear differential equation. In UQT the quantization of particles on masses appears as a subtle consequence of a balance between dispersion and nonlinearity, and the particle represents something like a very little water-ball, the contour of which is the density of energy. The quarks would be represented as diffraction maximums which appear after cutting spherical layer by a spherical harmonics, but these are the steps for future researchers. Many light particles have appeared in such model and there must exist some rules of selection. It seems to us, that these rules may appear in counting for very complex connections of spherical harmonics with various indices and tensor fields of high ranks. The authors are working intensively on this issue. Note that the whole observable spectrum is in the domain of initial indices of spherical harmonics and the selection presented consists of 169 particles. The probability of pure random coincidence of our results with experimental results is more than negligible, and we can state with no doubt that our theory is an important step forward in the theoretical construction of elementary particles' mass spectrum.

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